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Tugas Bab 2

(A)

1). $U_{xx} = 0$

$$\frac{\partial^2 u}{\partial x^2} = 0$$

$$\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x}\right] = 0$$

$$\frac{\partial u}{\partial x} = f(y)$$

$$u = x f(y) + g(y)$$

2). $U_{xx} + U_x - 2U = 0$

misal penyelesaian : $u = e^{y+mx}$

$$U_x = m e^{y+mx}$$
$$U_{xx} = m^2 e^{y+mx}$$

Sehingga, persamaan awal menjadi :

$$m^2 e^{y+mx} + m e^{y+mx} - 2 e^{y+mx} = 0$$

$$e^{y+mx}\left[m^2 + m - 2\right] = 0$$

karena $e^{y+mx} \neq 0$, maka:

$$m^2 + m - 2 = 0$$
$$(m+2)(m-1) = 0$$
$$m_1 = -2 \qquad m_2 = 1$$

Penyelesaian Homogen :

$$U = e^{y-2x} + e^{y+x}$$
$$= f(y-2x) + f(x+y)$$

3). $U_{xy} = 2x + 4y$

$$\frac{\partial^2 u}{\partial x \partial y} = 2x + 4y$$

$$\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial y}\right] = 2x + 4y$$

$$\frac{\partial u}{\partial y} = x^2 + 4xy + f(y)$$

$$u = x^2 y + 2xy^2 + F(y) + g(x)$$

4). $U_{xx} = \sin(2x - 3y)$

$$\frac{\partial^2 u}{\partial x^2} = \sin(2x-3y)$$

$$\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x}\right] = \sin(2x-3y)$$

$$\frac{\partial u}{\partial x} = \frac{1}{2}\left(-\cos(2x-3y)\right) + f(y)$$

$$u = -\frac{1}{2} \cdot \frac{1}{2}\sin(2x-3y) + x f(y) + g(y)$$

PUPD : $u = -\frac{1}{4}\sin(2x-3y) + x f(y) + g(y)$

10). $U_{xx} - U_{yy} + 2U_x - 2U_y = 0$

misal anggap $U = e^{y+mx}$

- $U_x = m e^{y+mx}$ $\qquad U_{xx} = m^2 e^{y+mx}$
- $U_y = e^{y+mx}$ $\qquad U_{yy} = e^{y+mx}$

$$\Rightarrow U_{xx} - U_{yy} + 2U_x - 2U_y = 0$$

$$m^2 e^{y+mx} - e^{y+mx} + 2m e^{y+mx} - 2e^{y+mx} = 0$$

$$e^{y+mx}\left[m^2 + 2m - 3\right] = 0$$

$$(m+3)(m-1) = 0$$

$$m = -3 \vee m_2 = 1$$

Penyelesaian : $U = e^{y-3x} + e^{y+x}$

$$= f(y-3x) + f(y+x)$$

(B)

1). $U_{xy} = 0$, $U_x(x,0) = \cos x$; $U(\frac{\pi}{2}, y) = \sin y$

$$\frac{\partial^2 u}{\partial x \partial y} = 0$$

$$\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial x}\right] = 0$$

$$\frac{\partial u}{\partial x} = f(x)$$

$U_x(x,0) = \cos x$

$f(x) = \cos x$ ... (1)

$$\frac{\partial u}{\partial x} = \cos x$$

$$u = \sin x + g(y)$$

$U(\frac{\pi}{2}, y) = \sin y$

$= \sin(\frac{\pi}{2}) + g(y) = \sin y$

$g(y) = \sin y - 1$

Diperoleh :

$U = \sin x + g(y)$

$= \sin x + \sin y - 1$

2). $U_{xy} = x^2 y$, $U(0,y) = y^2$, $U(x,1) = \cos y$

$$\frac{\partial^2 u}{\partial x \partial y} = x^2 y$$

$$\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial y}\right] = x^2 y$$

$$\frac{\partial u}{\partial y} = \frac{1}{3}x^3 y + f(y)$$

$$u = \frac{1}{6}x^3 y^2 + F(y) + g(x)$$

•> $U(0,y) = y^2$

$= \frac{1}{6}(0)^3 y^2 + F(y) + g(0) = y^2$

$F(y) = y^2 - g(0)$

$\Rightarrow F(1) = 1 - g(0)$

•> $U(x,1) = \cos y$

$= \frac{1}{6}x^3 + F(1) + g(x) = \cos y$

$\Rightarrow \frac{1}{6}x^3 + (1 - g(0)) + g(x) = \cos y$

$g(x) = \cos y - \frac{1}{6}x^3 - 1 + g(0)$

Sehingga diperoleh :

$U = \frac{1}{6}x^3 y^2 + F(y) + g(x)$

$= \frac{1}{6}x^3 y^2 + y^2 \cancel{- g(0)} + \cos y - \frac{1}{6}x^3 - 1 \cancel{+ g(0)}$

$= \frac{1}{6}x^3 y^2 + y^2 + \cos y - \frac{1}{6}x^3 - 1$